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MICROGRAVITY SEGREGATION IN BINARY MIXTURES OF INELASTIC SPHERES DRIVEN BY VELOCITY FLUCTUATION GRADIENTS

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608

ABSTRACT

We are interested in collisional granular flows of dry materials in reduced gravity. Because the particles interact through collisions, the energy of the particle velocity fluctuations plays an important role in the physics. Here we focus on the separation of grains by properties - size, for example - that is driven by spatial gradients in the fluctuation energy of the grains.

The segregation of grains by size is commonly observed in geophysical flows and industrial processes. Segregation of flowing grains can also take place based on other properties, e.g. shape, mass, friction, and coefficient of restitution. Many mechanisms may be responsible for segregation; most of these are strongly influenced by gravity. Here, we outline a mechanism that is independent of gravity. This mechanism may be important but is often obscured in terrestrial grain flows. It is driven by gradients in fluctuation energy.

In microgravity, the separation of grains by property will proceed slowly enough to permit flight observations to provide an unambiguous measurement of the transport coefficients associated with the segregation. In this context, we are planning a microgravity shear cell experiment that contains a mixture of two types of spherical grains. The grains will be driven to interact with two different types of boundaries on either sides of the cell. The resulting separation will be observed visually.

BACKGROUND

The size segregation of flowing or shaken grains is a commonly observed phenomenon in industrial processes and in nature. In many industrial processes a homogeneous aggregate is desired; in these, size segregation is undesirable. However, in the mining industry, segregation by size is exploited in some crushing operations. When observing natural grain deposits, grain segregation provides an indication of whether an aggregate of grains was deposited dry (larger grains above) or under water (larger grains below).

In systems that do not involve much agitation of the grains, several mechanisms that involve gravity have been identified as leading to such segregation. These include the preferential downward percolation of smaller particles in relatively slow inclined shear flows (e.g. Savage and Lun, 1988), the upward frictional ratcheting of large particles (e.g. Haff and Werner, 1986), and the preferential filling of space beneath larger particles by smaller particles in a system that is occasionally shaken (e.g. Rosato, Strandburg, Prinz and Swendsen, 1986, 1987).

In highly agitated flows there is a mechanism independent of gravity that is available to drive separation. This is associated with spatial gradients in the energy of the velocity fluctuations of the grains. Collisional interaction between and among different species of grains require that, in general, spatial gradients of concentration exist to balance spatial gradients of the particle fluctuation energy.

In sheared or vibrated collisional systems, gravity also influences mixtures of different size grains. Here, buoyant forces act to separate grains that differ in size and the local volume that they displace. The competition between buoyancy and gradients in concentration and energy may then

result in convection cells in which particles with different properties separate (e.g. Knight, Jaeger and Nagel, 1993).

In reduced gravity, such convection is suppressed and attention can be focused on the simpler balance between gradients in concentration and the gradients in fluctuation energy. Reduced gravity also eliminates the possibility that a collisional flow will condense into a slower, denser flow dominated by enduring contacts rather than by collisions.

Because collisions between grains inevitably dissipate energy, collisional granular shear flows are usually of limited extent in the direction transverse to the flow. One consequence is that shear flows are strongly influenced by their boundaries. Because grains, on average, slip relative to boundaries, a bumpy or frictional boundary can provide energy to the velocity fluctuations. However, because collisions between grains and the boundary dissipate fluctuation energy, there is a competition between production and dissipation.

In principle, it is possible to design the geometry of the boundary - for example, the size and spacing of regular bumps - so that the boundary either produces or dissipates fluctuation energy (e.g. Jenkins and Askari, 1993). This permits the control of the component of the spatial gradient of the fluctuation energy that is normal to the boundary. The gradients in fluctuation energy established by such boundaries may be exploited to drive the separation by size or other properties in a binary mixture of spherical grains.

We note here that microgravity makes the visual observations possible by permitting us to employ moderate rates of shear. On earth, the effects of gravity can be minimized by shearing so rapidly that the particle pressure overwhelms gravity. However, in this event, separation takes place too rapidly for visual observation, buoyancy and/or condensation associated with the centripetal acceleration must be accounted for, and the particles can be severely damaged.

Here we sketch the existing theory for collisional shear flows of binary mixtures of smooth, nearly elastic spheres (Jenkins and Mancini, 1987, 1989) and introduce the numerical simulations of the complete flow in the microgravity shear cell (Hopkins and Louge, 1991; Louge, 1994). The link between theory, simulations and experiments is provided by measurements of collision parameters in the apparatus described by Foerster, Louge, Chang and Allia (1994).

THEORY

Rather than providing here an exhaustive description of the complete theory, we introduce the governing equations for an unsteady, rectilinear shearing flow of a binary mixture of frictionless spheres, in which the gradient of the mixture velocity is vertical. These equations govern the time dependent response of an initially steady shearing flow to an increment in boundary velocity. This example provides an indication of how unsteadiness and gravity influence the theory for segregation.

In this case, the horizontal and vertical components of the balance of momentum for the mixture as a whole may be written as

$$\rho \dot{u} = S' \text{ and } 0 = -P' - \rho g ,$$

where ρ is the mixture mass density, u is the horizontal component of the mixture velocity, S and P are the mixture shear stress and pressure, g is the gravitational acceleration, and overdots and primes indicate derivatives with respect to time and vertical coordinate, respectively. The shear stress S is proportional to u' and the pressure P is proportional to the mixture fluctuation energy T . The coefficients in these expressions are given by Jenkins and Mancini (1989) for frictionless spheres as explicit, but extremely complicated, functions of the number densities, masses, radii, and coefficients of restitution of the two types of spheres.

Similarly, the balance of fluctuation energy for the mixture is

$$(3/2) \rho \dot{T} = -Q' + S u' - \rho \gamma,$$

where Q is the flux of mixture fluctuation energy, and γ is the rate of mixture dissipation of fluctuation energy due to the inelasticity of the collisions. The flux of fluctuation energy is proportional to T' and the rate of collisional dissipation γ is proportional to $T^{3/2}$. Jenkins and Mancini (1989) again provide the explicit forms of the coefficients. The presence of the collisional dissipation in the energy balance distinguishes the macroscopic system from its molecular counterpart.

Time dependent segregation of the spheres is described by an expression related to an approximate form of the difference between the balance of momentum for each species:

$$v_A - v_B = -n^2 D_{AB} (d_A + K_T T') / n_A n_B,$$

where v_A and v_B are the vertical components of the diffusion velocities, n_A and n_B the number densities of the two species, n is their sum, D_{AB} and K_T are, respectively, the coefficients of ordinary and thermal diffusion, and d_A is the vertical component of the diffusion force. It has the form

$$d_A = B_A P' + C_A T' + D_A n_A' + E_A n_B',$$

where the explicit forms for these and the diffusion coefficients are provided by Jenkins and Mancini (1989). When $v_A - v_B$ is different from zero, segregation is taking place; when $v_A - v_B$ vanishes, a steady balance between the gradients of fluctuation energy and gradients of species number density is attained. Segregation is influenced by gravity through the presence of P' in d_A .

Boundary conditions are obtained by calculating the collisional exchange of momentum and energy at the boundary. An expression for the slip velocity of the mixture results from balancing the sum of the collisional production of momentum with the mixture shear stress. The energy balance at the wall equates the normal component of the flux of mixture fluctuation energy to the working of the mixture shear stress less the sum of the species' collisional dissipation. Mancini's (1986) derivation of boundary conditions for bumpy, frictionless boundaries will be extended to include friction, and boundaries that produce fluctuation energy will be distinguished from boundaries that dissipate it.

In Figure 1 we show concentration profiles of two phases in a steady, fully-developed flow driven by the relative motion of identical, parallel, bumpy boundaries in the absence of gravity. These were obtained by Mancini (1986) as numerical solutions of the governing equations and boundary conditions for spheres of different diameters made of the same material. The boundary spheres were of the same diameter as the spheres of phase B and were assumed to be affixed in an hexagonal close-pack to two flat wall that were separated by a distance of 6.5 diameters of phase B. The spheres of phase A had a diameter 0.7 that of phase B, the average concentration of both species was taken to be 0.25, and the coefficients of restitution for all collisions were equal to 0.9. The full line in the figure is the concentration profile for a single phase of species B.

COMPUTER SIMULATIONS

The computer simulations are carried out to guide the design of the microgravity shear cell and inform the development of theory. The idea is to follow the dynamics of an ensemble of spheres interacting with the boundaries and among themselves through individual impacts. The

impacts are characterized by the three-parameter model that we establish for real spheres using the experiment described later.

Particle simulations are extensively used to perform numerical granular flow experiments. Chief among these simulations are the deformable particle simulations, in which the inter-particle forces are modeled during every impact (Walton 1983; Walton and Braun, 1986; Walton, Braun, Mallon and Cervelli, 1989) and the rigid particle simulations in which only the collisional impulses are modeled (Campbell, 1982, 1989).

Unlike earth-bound granular flows, bounded shear flows under low gravity do not readily condense into amorphous regions of negligible agitation. Without such condensation, the forces are primarily impulsive. In this case, Hopkins and Louge (1991) describe an efficient algorithm that permits simulations involving up to a hundred thousand spheres on a workstation of relatively modest size.

In their algorithm, collisions occur when a sphere overlaps slightly with another sphere or with the wall. The algorithm adjusts its time step periodically to ensure that the mean overlap is kept below a negligible tolerance. In addition, a search grid is superimposed on the flow domain to permit fast identification of near neighbors. Because this method makes it superfluous to maintain a list of future impacts, its computing time is merely proportional to the number of spheres N , unlike other algorithms that grow as $N \ln N$ or even N^2 . Figure 2 is an example of a microgravity shear cell geometry that we contemplate.

ACKNOWLEDGMENTS

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REFERENCES

- Campbell, C S, 1982 Shear Flows of Granular materials. Ph.D. Dissertation, California Institute of Technology, Pasadena, California.
- Campbell, C S, 1989 The stress tensor for simple shear flow of a granular material. *J. Fluid Mech.* 203, 449-73.
- Foerster, S, M Y Louge, H Chang and K Allia 1994 Measurements of the Collision Properties of Small Spheres. *Phys. Fluids* 6, 1108-15.
- Haff, P K, and B T Werner 1986 Computer simulation of the mechanical sorting of grains. *Powder Tech.* 48, 239-45.
- Hopkins, M A and Louge M 1991 Inelastic Microstructure in Rapid Granular Flows of Smooth Disks. *Phys. Fluids A* 3, 47-57.
- Jenkins, J T, and Askari, E 1993 Rapid granular shear flows driven by identical, bumpy, frictionless boundaries. In *Powders and Grains 93* (C Thornton, Ed.) Balkema, Rotterdam, pp. 295-300.
- Jenkins, J T, and F Mancini 1987 Balance laws and constitutive relations for plane flows of a dense, binary mixture of smooth, nearly elastic disks. *J. Appl. Mech.* 109, 27-34.
- Jenkins, J T, and F Mancini 1989 Kinetic theory for binary mixtures of smooth, nearly elastic spheres. *Phys. Fluids A* 1, 2050-7.
- Knight, J B, H M Jaeger and S R Nagel 1993 *Phys. Rev. Lett.* 70, 3728-31.

- Louge, M Y, 1994 Computer simulations of rapid granular shear flows of spheres interacting with a flat, frictional boundary. *Phys. Fluids* 6, 2253-69.
- Mancini, F, 1986 Transport Theory for Dense Binary Mixtures of Smooth, Round, Nearly Elastic Particles. Ph.D. Dissertation, Cornell University, Ithaca, NY.
- Rosato, A, K J Strandburg, F Prinz and R H Swendsen 1986 Monte Carlo simulation of particulate matter segregation. *Powder Tech.* 49, 59-69
- Rosato, A, K J Strandburg, F Prinz and R H Swendsen 1987 Why the Brazil nuts are on top: size segregation of particulate matter by shaking. *Phys. Rev. Lett.* 58, 1038-40.
- Savage, S B and C K K Lun 1988 Particle size segregation in inclined chute flow of dry cohesionless granular solids. *J. Fluid Mech.* 189, 311-35.
- Walton, O R, 1983 Particle-dynamics calculations of shear flow. In *Mechanics of Granular Materials - New Models and Constitutive Relations*, (J T Jenkins and M Satake, Eds.) Elsevier, Amsterdam, pp. 327-38.
- Walton, O R, and R L Braun. 1986 Viscosity, granular-temperature, and stress calculations for shearing assemblies of inelastic, frictional disks. *J. Rheol.* 30, 949-80.
- Walton, O R, R L Braun, R G Mallon and D M Cervelli 1989 Particle-dynamics calculations of gravity flow of inelastic, frictional spheres. In *Micromechanics of Granular Materials* (M Satake and J T Jenkins, Eds.), Elsevier, Amsterdam, pp. 153-62.

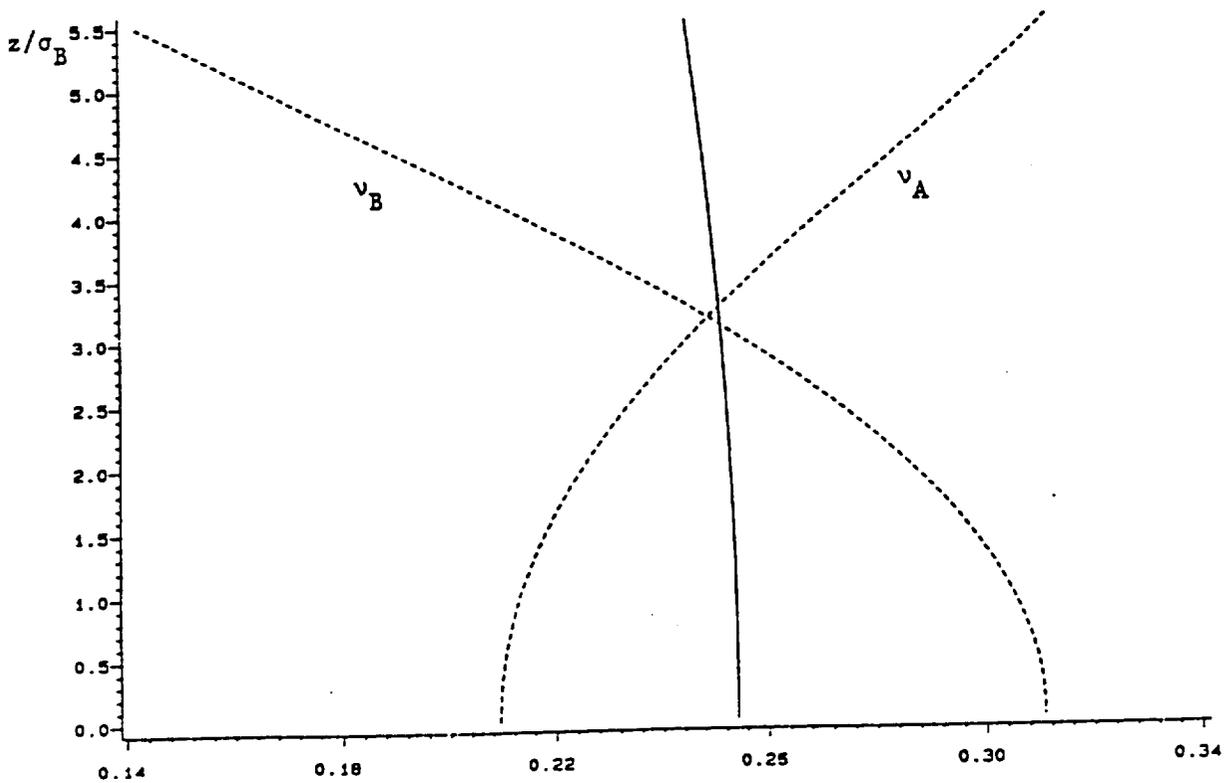


Figure 1. Concentration profiles of two species in the upper half of a shear flow driven by the relative motion of identical, parallel, bumpy boundaries in the absence of gravity.

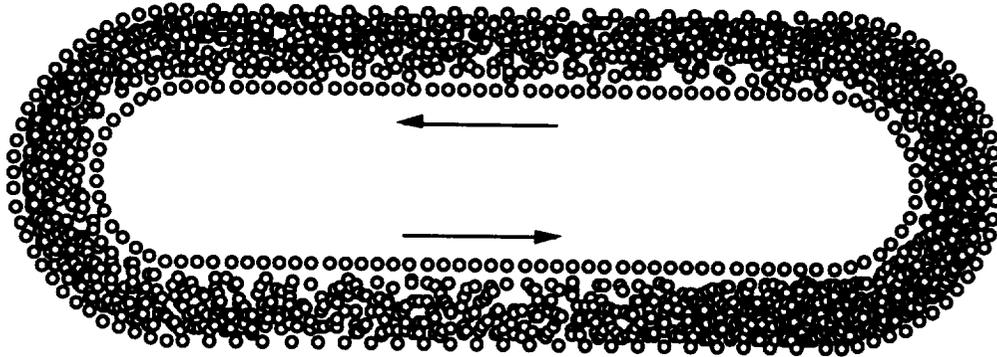


Figure 2. Snapshot from a numerical simulation of a microgravity shear cell involving identical boundary and interior spheres at an overall solid volume fraction of 30%. The inner boundary moves in the direction shown. The size of interior spheres is reduced for clarity.